Algebra Word Problems

Goals for this chapter

1. Learn a method for approaching word problems
2. Learn how to translate word problems into regular algebra problems
3. Learn how not to make word problems harder than they are

Why Worry About Word Problems?

> If half the contents of the smaller of two full pitchers of water were poured out it would contain one third of the contents of the larger; however, two of the smaller pitchers, if full, would contain five fluid ounces more than one of the larger. How much water would it take to fill both the smaller and larger pitchers, if empty?

Word problems – they drive some test takers crazy. Mathematically, however, they don't require any more knowledge than what we did in the previous chapter. That is, if you can solve equations you know all the math skills tested in word problems (in later chapters we'll see word problems based on math concepts, such as percents and probability, which we didn't learn in the **Algebra** chapter, so don't feel that you're being slighted). Why is it that people who have more or less mastered basic algebra, who can do every question from the **Algebra** chapter, can't handle word problems?

What's the difference between word problems and straightforward algebra problems? Both require algebra skills, but word problems require you to translate the words on the page (or on the computer screen) into equations which you can solve. This is the skill that we will learn in this chapter. The method we'll learn will work on any problem you come across (even on problems testing more advanced concepts, as we'll learn in later chapters). Also, we'll discuss ways people make questions more difficult than they have to be, and how to avoid this.

Has This Ever Happened to You?[1]

You read through a word problem. By the time you get to the end, where they ask you the question (e.g. "How much water would it take to fill both pitchers?"), you say, "I don't know how to do this?" and either give up, or start reading the question over again.

This is a common experience, and is a serious problem. What's wrong? Well, one of two things happens, as I said: either the test taker gives up, meaning that they miss the question, or they re-read the question, meaning they waste time. We don't want to do either – certainly you don't want to miss a question if you don't have to, and we both know that we don't have much time to waste on the GMAT.

Typically, the test taker actually knows how to do the problem, they just don't realize it. What's wrong with what they are doing? How could they have avoided this experience? Let's think about what it takes to answer a question on the GMAT.

To answer any question we must go through several steps. Can we do step 2 before we have done step 1? Of course not. Do we *need* to have done step 2 before we can do step 1? Again, of course not. <u>The key to doing multi-step tasks is completing each step before</u>

[1] This assumes you've done some word problems fairly recently. If you haven't, then trust me, this happens all the time.

going on to the next step. In order to do this well, you must give each step your complete and total attention. This means that you can't worry about what the next step is going to be – you'll get there when you get there. Remember what I said in the **How to Take the GMAT Math Section** about multitasking. Multitasking is a big no-no, because when you split your attention between more than one task, you don't do any of them as well. When you are doing a multi-step problem, if you are thinking about step 2 while you are doing step 1, you aren't going to do step 1 very well, and that might mess up the whole task.

How do this apply to word problems? Each sentence in a word problem represents at least one step in solving the problem. At the very least, it gives you information you need, and digesting that information is a step. In addition, the sentence may contain something, like an equation, which you need to solve, and that will be an additional step. You need to read and digest each sentence before you go on to the next one. If you read ahead before you've finished the steps the current sentence represents then you are wasting time and mental resources. You are thinking about more than one thing at a time, and some of what you are thinking about (the next sentence) is stuff that you can't do yet, because you haven't finished the sentence you are on.

Method for Doing Word Problems

Read one sentence (sometimes only a part of the sentence). Do everything you can to it. Then go on to the next sentence.

Now, this is the method you will use for *every single* word problem you ever do. It's not the only thing you need to know about word problems, however. There are a couple of additional skills you need to know; the big one is how to translate words into math. Let's do a problem now to illustrate the method, and in the course of the problem I'll also show you how to translate.

Example 1

> Bob buys two shirts and a jacket for $130. Sam buys a shirt and jacket for $100. How much does a shirt cost?

For those of you who are already decent at word problems, this may seem like a very easy question. Because it is a very easy question, you may not feel that it is necessary to go through all the steps I am about to show you in solving it. Not doing so, however, would be a mistake.

Your goal is to get a good GMAT score. In order to do so, you need to get a lot of questions right, both easy and hard questions. When you tackle the hard questions, you are going to need to have your skills well ingrained. The only way to do this is to practice them over and over again, so that using them is a matter of habit and doesn't require conscious thought. If, however, you don't use these skills on easy questions, they won't be as strong when you do hard questions. Take even the easiest question seriously and do it exactly the right way – it is an opportunity to build good GMAT habits.

OK, first step is to read the first sentence. "Bob buys two shirts and a jacket for $130." We've read it, and it sounds important. In fact, it sounds like something we should write down. How do we know? Well, the GMAT never gives us useless information (on problem solving questions). We aren't going to be able to remember these numbers, so we should write them down.

How do we write this down? We are going to write this down as an equation. How do we know? An equation is a comparison between values. It tells you that some value is the same as some other value. Here we see that the cost of two shirts and a jacket is the same as $130, so we have an equation. We now have to take the words from the page and translate them into parts of an equation.

There is a science to making equations out of English. It's fairly simple – you go word by word through a sentence, asking yourself what mathematical expression each word stands for. There is a limited number of options. Words can stand for numbers, either directly (e.g. "two" stands for 2) or as variables, words can stand for operations (+, - , x, or ÷), or words can stand for the equal sign. There are a few other options, but we'll learn them in later chapters.

Let's go through this sentence word by word. Does "Bob" mean anything mathematically? Does Bob refer to any number or any operation? No, so this will not go into our equation. Same with "buys" – buying is not adding, subtracting, multiplying or dividing. How about "two"? *This* is something we want to put in our equation. "Two" stands for 2; write it down. Our equation so far looks like:

$$2$$

The next word is "shirts"; Bob is buying two of them. How do we write down that we have two shirts? We already have the 2, so we just need to note that these are 2 shirts, specifically. We'll use a "s" to stand for "shirts".

$$2s$$

The word "and" means +, because when you add you put things together, exactly the way "and" does.

$$2s +$$

Now we see that Bob buys "a jacket" as well. Since this is one jacket, we'll write down "j" for jacket, but we won't write down a 1 (in algebra, a variable without a number is the same as 1 of that variable).

$$2s + j$$

The word "for" tells us that we are about to get the total cost of the items here; this is what these items equal.

$$2s + j = \$130$$

A final important step is: every time you write down a variable, you must write someplace what that variable stands for. Variables always stand for numbers. Here we have "s" and "j". "s" doesn't stand for "shirts", because "shirts" is not a number; "s" stands for *the price of shirts*. It's the same for j. Write down "s = price of shirts, j = price of jackets."

$$2s + j = \$130$$

s = price of shirts
j = price of jackets

And there we go – we have an equation. Whenever we have an equation, we should ask if we can solve it, and if we can we should. We can't solve this one, though, because we have too many variables. Since we can't solve it, there is no point staring at it – let's just move on to the next sentence.

Once again, "shirt" is s, "jacket" is j, "and" means +, "for" means =. We now have:

$$2s + j = \$130$$
$$s + j = \$100$$

Now we have two variables and two equations. What does this mean? It means we can solve the equations. Should we? After all, we haven't read the last sentence, the question sentence, yet. Maybe solving these equations will be a waste of our time.

But it won't be. Again, the GMAT never gives you useless information. If you are given equations which you can solve, the GMAT wants you to solve them. Solve them before you do anything else; in the long run it'll save you time and energy, because it will free your mind for thinking about the next step without the worry about what the equations tell you.

So go ahead and solve these equations on your own. When you are done, keep reading and I'll show you how I solved them.

Alright, ready? Here goes. I want to solve for one of the variables and substitute into the other equation. I'll solve the second equation for j. Why? Well, first of all, the second equation is going to be easier to solve. Second, if I solve for j I can just plug that into the first equation without doing a lot of multiplication, so the substitution will be easier as well. Would it kill me to solve for s? Not at all; it rarely makes a huge difference which variable you solve for, but you should generally pick the one that seems easier (without putting a lot of thought into it, which would cost more time than it saves).

So, solving the second equation:

$$s - s + j = 100 - s$$

$$j = 100 - s$$

Now I'm going to substitute this into the first equation.

$$2s + (100 - s) = 130$$

$$2s + 100 - s = 130$$

$$s + 100 = 130$$

$$s + 100 - 100 = 130 - 100$$

$$s = 30$$

Now, look back at where we wrote down what the variables stood for. It says "s = price of shirt" so we've figured out that a shirt costs $30. Now we read the last sentence of the problem. It asks us how much a shirt costs. That's exactly what we've discovered, so we're done. How about that?

It Seems Too Easy

Now, here's where you might be feeling some resistance. Notice that we got the right answer *without even reading the question*. Many students at first feel like they *have* to read the whole question, otherwise they'll waste time doing unnecessary work. This just isn't the case, however. Again, every piece of information the GMAT gives you is going to useful at some point in the problem. If the test presents you with an equation, it wants you to solve that equation; solving it sooner, rather than later, is always going to be a good thing. The experience we had on the previous question – reading the question last, only to find that we had already answered it – is going to be a common one due to the nature of the test.

But you might not believe me. After all, I wrote that previous question, so maybe I stacked the deck to make it easy in that way. Let's look at a variation on the problem – same information, different question – and see how it works.

Example 1b

> Bob buys two shirts and a jacket for $130. Sam buys a shirt and jacket for $100. How much would Fred pay for three shirts and two jackets?

OK, go ahead and use your work from the previous example, and solve this one. I'll give you a minute or so.

Alright, done? You should have got $230, and you should have seen that the work you did solving for s was totally necessary to answering this question, even though it's a completely different question. If you didn't, I'll show you what you should have done in a second. The point is, its hard to imagine a question they could ask where solving for s *wouldn't* be helpful. What if they'd asked for the price of a jacket? Knowing the price of a shirt, we can very easily find the price of a jacket.

Here's how you should have solved this one. The question asks for the price of three shirts and two jackets. You already know the price of shirt ($30), so multiply by 3 to get the price of three shirts: $90. How do we find the price of a jacket? Well, we know that a shirt and a jacket together costs $100 (that's from the second equation). Plug in $30 for s and solve for j. You should get that j = 70. Since one jacket costs $70, two cost $140; that, plus the three shirts, gives you $230.

Let's See Some More Examples

Now that you have an idea of how to do word problems, lets look at another example and see how other math concepts (besides addition) are tested.

Example 2

> John's earnings this week are $20 less than his earnings for last week. The total amount he earned over the course of both weeks would be just enough for him to buy a stereo costing $620 (including tax). How much did John earn this week?

First step first: read the first sentence. It is an equation, because it is comparing two values – John's earnings for this week and his earnings for last week. Go through word by word, translating into math. Go ahead, do it yourself, I'll give you my answer in the next paragraph.

OK, how did you do? You should have got something like this (although you probably used different variables):

$$t = L - 20$$

t = earnings for this week
L = earnings for last week

How did I get this? It's says "John's earnings this week"; this is some number, so it should get a variable (variables always stand for unknown numbers). I called it "t" for "this week". The next word is "are"; <u>words which means "to be" usually stand for the equal sign</u>. This is because the equal sign tells you what something *is*. The "$20" obviously just means "20", and "less" means that we should subtract; we subtract from his earnings from last week, which I called "L".[2]

Important Note:
If you see that one value is some amount less than another value, you should subtract. In fact, they mean the you should subtract the amount from the second value. In other words, if I say that my height is 4" less than Roberts, we know that if we subtract 4" from Robert's height, we get mine. <u>Always subtract from what follows the "than"</u>.

For example:

3 second less than Amy's time would give you Jessica's time for the race.

Subtract 3 from *Amy's time*, because it follows the "than":

$$a - 3 = j$$

Why is this the case? Well, if I say "x is so much less than y", then y is bigger than x, right? And how do I make x and y equal? I have to take something away from the *bigger one*, right? Whichever follows the than is the larger one (because the other one is *less* than it), so I always have to subtract from it.

We have two variables, but only one equation, so we can't solve. Let's go on to the next sentence. Again, do it yourself before reading the next paragraph.

The next sentence contains an important word: "total". We get totals by adding things up, so <u>"total" tells you that you are going to use addition</u>. The sentence talks about the total of what John earned over the two weeks; thus, we add t and L. If it allows him to just afford the stereo, the total must be equal to the amount of the stereo. So we get

$$t + L = 620$$

Now we have two variables and two equation, and we can solve. Notice again that we are solving before we read the last sentence of the question.

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[2] Notice that I use an upper case L, rather than a lower case one. This is because the lower case L looks too much like a 1. I might get confused later on. This type of attention to detail can save you points here and there, and its not too hard if you start making a habit of it right now. For example, I cross my z's when I write them (**EXAMPLE**), otherwise they look too much like 2's. I encourage some students to always use upper case A's, because lower case a's, if written quickly, can look like 9s (as can lower case G's). Watch out for the types of little mistakes you make, and notice if little changes like this would stop them.

The first equation is already solved for t, so substitute that into the second equation.

$$(L - 20) + L = 620$$

$$2L - 20 = 620$$

$$2L - 20 + 20 = 620 + 20$$

$$2L = 640$$

$$\frac{2L}{2} = \frac{640}{2}$$

$$L = 370$$

Now that we have solved, read the question. Oh, no! It asks for t! What shall we do? Well, we know that t = L -20, so plug our value for L into that equation. We get

$$t = 350$$

And we're done. Notice here that getting a value for L first is actually easier than getting one for t first (even though t is what the question asks for). This is because to get a value for t first we'd have to solve one of the equations for L and then substitute; t, on the other hand, has already been solved for. Thus, reading the question and doing what it suggests is actually *slower* than not reading the question and doing the obvious.

Summary on Translation

Most of the words you'll see on the test have a direct math equivalent. Let's look at a few right here.

English Word	Math Equivalent
and	+
less than x	x −
more than x	x +
is/would be/will be/ was	=
total	something + something =

We'll see more as we get further into the chapter.

Data Sufficiency

I hope you remember what you learned about data sufficiency from last chapter, because it's going to be extremely useful in this chapter. Let me test you:

Q: What's the first thing I look for to see if a statement is sufficient?

A: I count the number of variables and the number of equations. If they are equal, then the statement is going to be sufficient.

Q: What do I do if the number of equations is less than the number of variables?

A: Then I do some math, because I might be able to solve anyway.

Alright, well done. Data Sufficiency word problems are mostly just translation exercises. Once you've translated the words into math (and written it down), you just apply what you know about algebra. In a way they are easier than Problem Solving questions, because you don't actually have to solve the problem.

Example 3

Some of Ivan's money is invested in stocks, and the rest is invested in bonds. How much of Ivan's money is invested in stocks?

1) Ivan has a total of $10,000 invested.
2) Ivan has three times as much invested in stocks as in bonds.

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Take a minute to figure this one out. I'll give you a hint: remember to do each statement by itself, without looking at the other statement. Then, if they are both insufficient, put them together.

Solution to Example 3

The first sentence tells us that Ivan only has stocks and bonds, nothing else. Let's create some variables:

s = amount invested in stocks
b = amount invested in bonds

The question asks how much is in stocks, so we write down

$$s = ?$$

This is a reminder as to what we are trying to solve.

In statement one we see the word "total". When we see "total", we know that we are adding things up. Here, what are we adding? Ivan only has two types of investments, so we add these to get his total investment.

$$s + b = 10000$$

Here we can't solve for s, because we have two variables but only one equation. Mark it as insufficient, and go on to statement two.

This is an equation as well, because it compares the amount in stocks to the amount in bonds. This sentence might be a bit confusing. It says Ivan has "three times as much invested"; "three times" clearly means that we multiply something by 3, but what? Read the sentence again and ask yourself, which does Ivan have more of, stocks or bonds? Stocks are three times as much as bonds. Since Ivan has more in stocks, in order to make stocks and bonds equal, we have to make bonds bigger – we have to multiply it by 3.

$$s = 3b$$

Another way of seeing this is by seeing that the amount in stocks are three times the amount in bonds; *that* way of stating the sentence has "are" in it, which means =.

Again we have two variables but only one equation. This is also insufficient.

We have to combine the two statements. Each statement gives you a new equation; together you have two equations and two variables. Thus, you can solve for s; the answer is C.

We've learned another math term: "times" means we have to multiply.

English Word	Math Equivalent
and	+
less than	-
is/would be/will be/ was	=
total	something + something =
times	x

Age Problems

Let's briefly recap what we've learned about word problems:

1. Read word problems sentence by sentence, doing all necessary work before moving on to the next sentence
2. Sentences which compare two values are equations
3. Memorize English-to-math equivalencies, and translate equations word by word
4. Always write down what your variables stand for
5. Variables always stand for unknown number values

Now that you understand these ideas and how to apply them you can tackle any word problem based on the algebra we've learned so far. However, there are some types of word problems that occur over and over again on the GMAT; these problems have their own nuances, which will be helpful to know when you see them on the test. Take a look at the following problem:

Example 4

> John is 10 years older than Mary. If each were twice as old as they are now, John would be three times as old as Mary. How old is John?

This is what I call an age problem. Age problems are technically no different from any other word problems – you make equations, label variables, and solve. However, there is one small difference. Ages change in ways that normal variables do not. That is, someone's age doesn't stay the same, where most variables do. Every year, someone's age will increase. Makes sense, right? Well, we need to build that into the way we handle the problems.

Let's go through this sentence by sentence, the way that we are supposed to. Read the first sentence and ask yourself, "Is this an equation?"

If you answered yes, you are right. It's an equation because it compares two values (John's age and Mary's age). Go through word by word; "John" means John's age, and since we don't know how old John is, this will be a variable. As always, "is" means "equals", and "older" means we are going to add to Mary's age. We get the following

j = John's age now
m = Mary's age now

$$j = 10 + m$$

Take a look at how I've labeled the variables. Notice the word "now". This is how we build in the fact that ages change with every year that passes. How old will John be in 5 years? He'll be 5 years older than he is *now*, so "t + 5". Same for Mary; she'd be "m + 5".

Take a look at the next sentence. It too is an equation. We are going to double each of their ages. What is John's doubled age? It is 2j (two times his age now). Mary would be 2m (twice her age now). Let's go through this word by word. When they say, "If each were twice as old…" they are just telling you that for the rest of the sentence they are talking about their doubled ages, not their current age. <u>If the sentence didn't specify, it would be talking about the age after the change</u>. So, when they say "John" they are referring to John's doubled age, not his age now, so this is "2j"; "would be" means "equals"; "three times is obvious; and "Mary" refer's to Mary's doubled age (2m), not "m". We get:

$$2j = 3(2m)$$

One very common mistake is for students to say "2j = 3(2(m + 10)). This comes from thinking too much, and not just using what is right in front of you. Students want to use what they *think* they know – they think they know something fancy about Mary (that she is "m + 10") – rather than what they *know* they know (that Mary is "m" years old). In reality, "m + 10" is *John's* age, so if you had put it for Mary's age you would be saying that John is 3 times as old as he is. Put that way, it's clearly wrong. The point is this: when trying to figure out what variables to put into an equation, or how to use them, make sure you refer back to the labels you have written down; if you do, you won't misuse your variables (in this case, you'll realize that twice Mary's age is just 2m).

At this point we realize that we have two variables and two equations, so we can solve. I want to reiterate that we *have not read the question yet*! That is, we don't know what they want us to solve for, and that's ok. We'll solve for whatever is easiest. Here's what we have so far:

$$j = m + 10$$
$$2j = 3(2m)$$

In the first equation, j is already solved for. Let's just go with the flow: substitute that into the second equation. We get

$$2(m + 10) = 6m$$

I assume you can take it from here. Go ahead, I'll wait.

Done? Ok, you should get that m = 5. Now we read the question. Oh, no! It asks "How old is John?" But, it's very easy to figure that out, now that we have Mary's age. John is 10 years older, so he's 15. And we are done. This is actually easier than if we had read the question and solved for j first. Always solve for whatever looks easiest; if it ends up not being what you are asked for, it'll still make answering the question easier.

Two lessons: first, when dealing with age problems, make sure when you label your variables with "now" where appropriate.[3] Second, solve for the easiest variable first, then read the question.

Example 4

Tyler is 5 years older than Megan. How is old is Megan?

1) If Tyler were 3 years younger, he would be twice as old as Megan is now.
2) In 3 years, Tyler will be twice as old as Megan

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

Remember, set Data Sufficiency questions up just like Problem Solving questions. You should have written:

t = Tyler's age now
m = Megan's age now

$$t = 5 + m$$
$$m = ?$$

Now let's look at Statement 1. This is a little different from what we've been doing. It changes Tyler's age, but keeps Megan's age as it is now: m. You should get the following equation ("younger" means subtract from his current age):

$$t - 3 = 2m$$

This plus the original equation gives us a total of two equations; we have two variables, so this is sufficient. On to Statement 2.

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[3] Some of you might worry about this. What if the age we are given is *not* the age of the person now? Then label it appropriately. If you are told "5 years ago Fred was f years old" you can write "f = Fred's age 5 years ago."

In three years, Tyler and Megan's ages will both have changed. Both will be 3 years older than they are now, so they will be "t + 3" and "m + 3" respectively. When the sentence says "he", it is talking about Tyler's age in 3 years, and "Megan's age" means "Megan's age in 3 years". If they had wanted to talk about Megan's age now, they would have said, "In three years Tyler will be twice Megan's current age," or something like that. Our new equation is

$$t + 3 = 2(m + 3)$$

Again, this gives us two equations total (not three, because we look at each statement individually). Since we have two variables, this is sufficient. The answer is D.

Example 5

> Susan's bag has only pens and books in it. Pens weigh 3 ounces each and books weigh 12 ounces each. Everything in her bag together weighs 39 ounces, and she has seven things in her bag. How many pens are in the bag?

This is a type of question which you are very likely to see on the GMAT. Once you've done this sort of question it's pretty easy to recognize them in the future, and to know exactly how to approach them. But the approach takes some explaining; I'm going to walk you through a long process that you won't actually go through on the GMAT, but that should make the question understandable. Then we'll go through how to recognize an approach these questions on the GMAT.

The first sentence is nothing to write down. It just tells us that there are only two things to think about – pens and books – and we don't have to worry about anything else (makeup, money, paper clips, etc.). Now the second sentence tells us a little about pens and books. The temptation is to write: "p = 3" and "b = 12". Don’t do this.

Variables stand for things you don't know. But you *do* know how much pens and books weigh – they weigh 3 and 12 ounces (respectively). So why create new variables? Instead, just write down what you are told:

pens weigh 3
books weigh 12

The next sentence has two equations (one about total weight and one about total number), so let's just read up until the comma and stop. We are told "Everything in her bag together weighs 39 ounces." When they say everything, what do they mean? She only has pens and books, right? So they are really talking about the total weight of all the pens and all the books. So we know

total weight of pens + total weight of books = 39

This is not an equation we'll use on the GMAT, but it'll help us understand what we eventually do. The next part of the sentence tells the total number. As I said before, "total" means we are adding up things. What are we adding here? We are adding up the number of pens and number of books.

number of pens + number of books = 7

This is also not an equation we'll use on the GMAT. We're getting close, though.

Look at our two equations. How many variables do we have? We've got 4 (weight of pens, weight of books, number of pens, number of books). That's too many. How can we simplify this? Well, how do we get the total weight of the pens? Think about it before you go onto the next paragraph.

We know one pen weighs 3 ounces. If we knew the number of pens we have, we could multiply that by 3 and get the total weight of the pens. The same for books (except they weigh 12). So the 1st equation can be written as

3(number of pens) + 12(number of books) = 39

Now we only have two variables, and we've got two equations. This is something we can handle. Let's make simpler variables here, and re-write our equations.

p = number of pens
b = number of books

$$3p + 12b = 39$$
$$p + b = 7$$

I trust that you can solve from here. Go ahead. If you need help, read the next paragraph.

Solve the second equation (because it's much simpler) for either p or b (I prefer p, because the multiplication is easier when you substitute it into the first equation). Then substitute that into the first equation and solve for whatever variable you have left.

Here's the GMAT lesson. <u>You might see a question talking about the total number of things you have, and also some value associated with each</u>. Here the value is weight, but it could be cost, length, brightness, or any other number. <u>You'll create two equations: one where you add the number of each thing to get the total number, and the other where you multiply the number of each thing by its value, and add these two totals to get the total value</u>. After that its just algebra.

The answer here, by the way, is 5; there are 5 pens.

Example 6

If Enrico has only \$20 bills and \$5 bills in his wallet, how many are \$5 bills?

1) He has \$75 total in his wallet.
2) If he spends \$10, he'll have 4 bills left in his wallet.

A. Statement (1), by itself, is sufficient to answer the question, but statement (2), by itself, is not.
B. Statement (2), by itself, is sufficient to answer the question, but statement (1), by itself, is not.
C. Statements (1) and (2) taken together are sufficient to answer the question, although neither statement by itself is sufficient.
D. Either statement by itself is sufficient to answer the question.
E. Statements (1) and (2) taken together are not sufficient to answer the question, nor are they sufficient to answer the question by themselves.

This problem is the same sort as the one we just looked at. We know this because we have two sorts of things (the $20 and $5 bills) which have different values ($20 and $5), and, finally, the question is talking about the number of bills we have. So we know that we are eventually going to set up an equation about the total dollar value we have, and the total number of bills we have.

Indeed, Statement 1 tells us about the total dollar amount. Set up an equation like in the previous question: value(number) + value(number) = total value.

f = number of $5 bills
t = number of $20 bills

5f + 20t = 75

This is insufficient: two variables, one equation.

Statement 2 looks a little tricky. It talks about the total number of bills, but only after we spend $10. There are several seemingly obvious ways we can try to set up this equation, but they are all wrong. Here are some *wrong* ways to do this:

wrong 5f + 20t = 65
wrong 5f + 20t = total – 10
wrong 4 + 10 = f + t

The first example is wrong for two reasons. First, it is using information from Statement 1, that we have a total of $75. Remember, you have to look at each statement by itself first. Second, 5f + 20t adds up to 75, not 65, so this is factually false. The number of $5 and $20 is different now, because we've spent $10, so we can't use the same variables. This is why the second equation is wrong; it's got the right idea, but it uses the same variables as Statement 1, when, in fact, at least one of "f" or "t" has to be different. The third equation is trying to say something about the total number of bills, but $10 is an amount of money, not a number of bills.

The third equation is close, though. Think about what you are being told. Enrico has spent $10. That means f or t have changed. What kind of bills did Enrico *have* to spend? He had to have spent $5 bills; you can't make $10 out of $20 bills. This means he spent 2 bills, so he must have had 6 before. So now we know

f + t = 6

This by itself is insufficient (2 variables, one equation). But, when combined with Statement 1, they are sufficient. The answer is C.

GMAT lesson: <u>in these value questions, you can only have a whole number of each type of thing</u>. <u>You can sometimes use that information to make some deductions about what number of each type of thing you have to have</u>. So, in these questions, don't just make equations, think about what you are being told means. That's one of the reasons we want to recognize these questions: they require a little additional thought, and we only want to do extra work where it will help us.

Picking Numbers

Brian, maybe you should have talked about picking numbers in the Algebra chapter? After all, there are some Algebra questions that are easier if you pick numbers… Hmmm… well, look in the GMAT book and see if you can find some.

Example 7

A salesperson sells cars, either SUVs or economy cars. The salesperson receives a bonus of *b* dollars for every SUV sold over a certain number, *a*. If the salesperson sells *c* cars, *e* of them economy cars, what is their total bonus?

A. bc - be
B. (c(e + a))b
C. ba - (bc - be)
D. bc - be - ba
E. b((c - a) + e)

As you read the question, notice that you are given a series of variables, but no number values. When you are given a variable in a question, rather than a number, skim the rest of the question to see if you are given any number values at all. <u>Any time you see a question with no numbers, you should approach the question by picking your own number values</u>. You can pick any number you want for each variable, but try and pick ones which make the question easy.

Pick a number for the bonus and for the amount of SUVs he has to sell; any numbers will do. <u>As you pick numbers, make sure you understand what they represent</u>.

b = $10
a = 5

The salesperson has to sell more than 5 SUVs to start getting a bonus. They will get $10 for every SUV after 5.

Make sure you understand what this means. If the salesperson sells 6 SUVs, he'll get a $10 bonus. If he sells 7, he'll get a $20 bonus. We know this because the question says he gets the bonus for "*every* SUV sold" over a.

Pick for c and e; c has to be larger than a for the salesperson to get a bonus. It's ok, however, if you picked a smaller number for c at first. When you do so, you'll soon notice that the salesperson gets no bonus, and that isn't nice. Generally, when you are picking numbers any number will do. However, sometimes certain numbers won't make sense. If you use common sense as you go you'll catch this as it happens, so don't worry about it too much.

c = 20
e = 10

So, the salesperson sells 20 cars; 10 of them are economy. If 10 are economy, how many SUVs have they sold? There are only SUVs and economy cars; if 20 cars are sold and 10 are economy, then 10 must be SUVs.

SUVs = 10

If they sell 10 SUVs, how many cars do they get a bonus for? Everything over 5 earns a bonus, so the salesperson gets a bonus for 5 SUVs. The bonus is $10 per car, so

total bonus = $50

Once you have gotten a result, you have to now pick the right answer. <u>If the answers have variables (as here), you must plug the numbers you picked into each answer and see which one gives you the proper result</u>. We substitute all these variables into the answers to see which one gives us $50.

A. 10(20) - 10(10) = 200 - 100 = 100

B. (20(10 + 5))10 = (20(15))10 = 300(10) = 3000

C. 10(5) - (10(20) - 10(10)) = 50 - (200 - 100) = 50 - 100 = -50

D. 10(20) - 10(10) - 10(5) = 200 - 100 - 50 = 50

E. 10((20 - 5) + 10) = 10(15 + 10) = 10(25) = 250

D is the correct answer (we didn't even have to look at E). That may have seemed like a lot of work to you. In fact, it is a lot of work, although it isn't more than two minutes worth of work. The nice thing is that it is *easy* work – just arithmetic. Doing this question any other way involves hard work – thinking – that you can't guarantee you'll do properly. Do the question this way and you will get the right answer.

Conclusion

This is everything I have to say about word problems in general. In the rest of the book we'll be looking at specific types of word problems – those involving Geometry, or Ratios, and so forth. Each type of word problem builds on what we covered in this chapter, as well as relies on knowledge of a specific subject. But it is crucial that you are comfortable with the basic skill we learned in this chapter: read problems one sentence at a time, writing down everything you are told as you are told it. As you write things down, do any appropriate work – simplify or solve equations, do any arithmetic – and don't worry about what the next step will be. You'll find that when you get to the actual question, most of the time you'll already have solved it, or at least have done the important steps leading up to solving it.